pupil in large classes are necessary to-day. Every pupil has a right to an opportunity and we must recognize that some get their earliest perception through one sense and some through another. Good devices often require long practice and great skill from the teacher. (3) Every teacher should, so far as possible, master a grade and the peculiar pedagogic difficulties the first time he teaches it; and should never let himself grow unduly rusty on any part of high school mathematics.

Good *teaching* is what our secondary schools to-day need rather than highly specialized scholarship or philosophical quibbling. Thus will the standard of our calling tend to rise to its true position—the noblest of the professions.

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MATHEMATICS FOR TRAINING AND CULTURE.*

BY W. H. METZLER.

To start a reform requires sufficient energy to overcome the inertia of long habit and custom and to get out of the ruts it is often thought necessary to pull with so much force that the wheels are far beyond the level track before the energy is expended or the fact realized.

The history of the teaching of mathematics shows a series of fluctuations from one point of emphasis to another, and all reform carries with it a good deal which in time is set aside as worthless. Some reformers see little or no good in anything of the present, but think because some things are not what they should be all should be upset and an entirely new structure built upon new foundations. True reforms seldom, if ever, come in this way. Certainly the reform in the teaching of mathematics will not, but much of the present and of the past will be found best for the future.

The spirit of the age and country affects much the spirit of the schools, so we find many thinking that the spirit of commercialism, so prevalent in this age and in this country, should guide our school life and teaching. This is the age of commercial and trade schools and I do not think we have enough of

*Read before the New York Section.

them, but I do not want all schools pervaded by the same spirit and aim we find for the most part in them. I feel quite sure that some of these schools would be improved by a modification of their spirit and aim. I am not as yet convinced that it is the duty of the state to support schools the object of which is to teach trades or direct means of making a livelihood. I would certainly maintain that this duty did not exist outside the large centers of population. It is the duty of the state to support schools whose object is the making of better citizens. To aim at the purely practical, commercial or utilitarian is to aim at the level of mere existence which is the level of the beast. To rise above this level the aim must be to reach the elements of training, culture and morals. The possession of these is what makes one citizen superior to another who lacks them. I believe that every individual, in so far as it is possible, should earn his own living, but a citizen that can make a living for himself is not necessarily a good citizen. Indeed he is not a good citizen unless he goes beyond this and makes a better man out of himself and helps others to be better, unless he leaves the world better for his having lived in it. Men should be judged by what they are and do, not by what they possess. Unless the millionaire uses his money for the betterment of mankind he is of no benefit to the world for possessing it. Unless the scholar uses his training for the same purpose, he likewise is of no benefit to the world because he possesses it. It is the duty of every one to get all the wealth he can honestly, and to obtain all the knowledge and training he can, but it is much more his duty to use these things for the benefit of mankind. It is the giving-out process that is more important than the taking-in. Now the emphasis of those who aim at the practical or utilitarian is on the getting rather than on the giving-out. To emphasize the taking-in process is to emphasize selfishness, but to emphasize the giving-out is to emphasize unselfishness and to build up true character which is necessary for good citizenship. Good citizenship means good character, good character means altruism; while utilitarianism tends towards selfishness. None of the truly great characters of the world were selfish. True greatness and selfishness cannot exist together.

The tendency seems to be to hold up before the youth of today, as examples worthy of emulation, men who have acquired

great wealth, while little or no reference to how they got it or to how they are using it. Not long ago I heard an educator say "We want to instill in our youth the ambition and spirit to get there." With this I have no immediate quarrel, but I want to know *how* my boy gets there and *what* he has got after he gets there. I want him to strive and to strive hard but it must be for something worth while. It must be for something he can use for the good of the world and not a mere possession of little use to him and of less to others.

The world needs more philanthropists, but it also needs to recognize that there are philanthropists other than those who give of their money. He is no less one who gives of his time, his education, or his very life for the good and betterment of others. Are there no philanthropists in the school room? The examples we hold before our pupils should possess less of the commercial and more of the altruistic.

Education means to lead out and not to feed in. It means the systematic development and cultivation of mind and heart. To cultivate is to prepare the soil for growth and development. To cultivate the mind is to improve and develop it by study and effort. Culture is the result or product of cultivation.

As for a vigorous physical life there must be food, exercise and rest in proper proportions and at proper intervals as long as such life is to last, so for a wholesome vigorous mental life there must be mental food, mental exercise and mental rest in proper proportions and at proper intervals. It is the business of the schools to prepare the student so they will be enabled to take these all through life, and not have their school days one long period of taking mental food, followed by another long period of mental rest.

To emphasize the practical and ignore training and culture will result in anything but a vigorous mental life. Such emphasis is like sowing seed upon uncultivated ground, in which case the birds of forgetfulness soon devour most of it and what little grows will not yield much fruit. The seeds of methods, principles, habits and morals must be sown on well-cultivated ground when they will yield some thirty, some sixty, and some an hundred fold, and the beauty of the crop is that it never grows less and is always ready. If there is no cultivation or culture there is no exercise or effort and where there is no effort there is no

development and where there is no development atrophy and death result.

In my judgment the most important work of every teacher is the moulding of character, for without that no one is fitted for any walk of life. Character is what counts even more than training. The two should grow together. In speaking of the college man as an engineer Mr. F. W. Taylor, in the *American Machinist* for October 7, 1909, says that for success, " Without the slightest question character comes first."

In this country and particularly in the large cities, with so many of foreign birth, where the schools must be responsible for so much of the pupils development if they are to reach higher than the mere plane of existence and of making a living, the training and culture elements are to be ever uppermost and must be constantly and for ever emphasized to attain the results necessary for better citizenship. Is it not a lack of culture and morals that makes the problem of the East Side in New York? If all the citizens of Denver were cultured and honest would Judge Lindsey be having such a fight as he is? Surely no one interested in our schools desires to introduce in them more of the spirit of commercialism or to intensify this spirit. The problem of the foreign element and good citizenship will be solved better and sooner by getting away from this spirit and emphasizing more the training and culture side of subjects.

I am inclined to think that there is a good deal of misapprehension as to the practical value of mathematics. Outside of the fundamental facts of arithmetic there are very few of the facts of mathematics put into direct use by people in general. These then are not the most important things, but the methods, principles, and the type of thought involved in mathematics are so universal and independent of time and place (as all civilized people have them) that no education can afford to neglect them. To be sure the facts of mathematics and its practical applications are of very great indirect importance to the race and of direct importance to a few individuals, but of vastly more importance to those in school are the modes of thought which it exemplifies. The practical side of mathematics is of value to the few while the culture side is of value to all. This in itself should be sufficient to decide where to place the emphasis.

All thinking results or should result in conclusions and our

thinking and actions are influenced by other conclusions. The facts of everyday life on which we base our reasoning are so many and so complicated that drawing conclusions is not an easy or simple matter. It is the business of the schools to train students to the habit of drawing correct conclusions and of reasoning accurately. No other subject in the school curriculum can do as much in this direction as mathematics if taught properly. Its mode of thought and its conclusions are of the same type as found in everyday life. Its conclusions are certain and proceed from the easy to the more difficult, and as a preparatory training to the difficult conclusions of life this is of great importance.

In the reasoning of everyday life we estimate the correctness and completeness of the facts and on the hypothesis that they are true draw our conclusions. In mathematics the student is taught to reason out the steps for himself, he must pick out the essentials, and at every stage there is an act of judgment and reasoning brought into play as to the next thing to be done and the best way of doing it. There must stand out clearly what is given and what is to be done, what are the tools to do it with and how they are to be used, and at the end it must be clearly seen that the conditions have been met. This must become a habit of the pupil's life and when so he has something with which the practical side has no comparison.

Those who advocate mathematics for purely practical purposes, to be consistent, should remove everything from the curriculum except those things which promise a direct reward from without themselves. This done there would be little left. Even play which finds its end within itself would be banished from the child life. Indeed it may be questioned whether it is not true, that, in harmony with the spirit of the times, the sports of the high schools and colleges of to-day have not shifted the end of these sports from within to without, and they are no longer ends in themselves but means to an end.

Let us hold up before our pupils then, ideals that will counteract this tendency rather than those which intensify and prolong it. Let us show them that many things are done for their own sake and that they should not be always looking for direct rewards for their labors. Yes let us even show them that some things should be done for the benefit of humanity. In order that you may see what others who have had good opportunity to

study the results of our school work have to say on this point, let me quote from Mr. J. G. P. Stokes, the philanthropist and settlement worker of New York city. He says: "Until recently it appears to have escaped public notice that this constant emphasis (given by the schools) upon the importance of personal success, unless safeguarded by suitable ethical training, tends subtly to the development of selfish propensities, that lead the individual to disregard or subordinate the interests of others, in the furtherance of personal ends; and that lead to unsocial attitudes, and to unfriendly rivalries and ill-feeling, and to wrong doing of every sort. The constant encouragement given to personal ambition or personal triumph and personal reward tends to develop a desire similar to that possessed by the criminal offender, who, in seeking his personal gratification, gives no proper regard or consideration to the relation of his acts or of his course to the welfare of others or to the welfare of the community."

I have elsewhere (*Journal of Pedagogy*, June, 1905) gone into some details as to the educational value of mathematics and have shown how the proper study and teaching of it develops every power and ability which characterizes the educated being. I will not therefore tire you to repeat these details here, but I would like to add some other points to those which I have already mentioned in this paper.

In mathematics more than in any other subject the student is enabled to discover for himself and to enter into the real spirit and enthusiasm of discovery. It is here that he can have, as nowhere else so early, mental activity for its own sake, in which case his stimulus comes from within, the source of all good stimuli. Its finished form makes it the model or ideal for all sciences. Mathematics has a large esthetic as well as ethical value. Poincaré says: "Mathematics has a triple end. It is to furnish an instrument for the study of nature. But that is not all. It has a philosophic end, and I dare say it, an esthetic end. . . . Those skilled in mathematics find in it a pleasure akin to those which painting and music give." Shellback says: "Who does not know mathematics and the results of recent scientific investigation, dies without knowing truth."

I know of no other subject which furnishes a more valuable training in logical thinking or in clear and concise expression,

and in fact, for training all students for general service, mathematics is surpassed by few if any subjects. In order to see how other countries look upon it let me give a few quotations from Europe. From the Prussian curricula: "For the secondary schools, the most important task of instruction in mathematics lies in a training of the mind which enables the pupil to use correctly in his own independent work the intuitions and knowledge which he has acquired. In all domains of this subject the aim must therefore be to attain a clear understanding of the theorems to be developed and their deduction, as well as practice and skill in their use."

From the Austrian curricula: "Instruction in mathematics has in general the important duty of coöperation in the development of the power of thought of the pupils, to lead them to the formation of independent judgment, to facilitate the understanding of the laws of nature, and no less than any other branch of instruction to cultivate the clear expression of thought in correct language.

"Consequently, such portions of elementary mathematics have been incorporated into the curriculum as have a recognized high culture value, and in an order corresponding to the progressive mental development and power of comprehension of the pupil. Incidentally, the selection has also to take into account the needs of practical life and the connection of mathematics with other fields of knowledge, notably with the natural sciences."

From the German Society for the Advancement of Instruction in Mathematics: "In the secondary schools mathematics should be a part of general culture and not contributary to technical training of any sort; it should cultivate space intuition, logical thinking, the power to rephrase in clear language thoughts recognized as correct, and ethical and esthetic effects; so treated mathematics is a quite indispensable factor of a general education in so far as the latter shows its traces in comprehension of the development of civilization and the ability to participate in the further task of civilization. Accordingly applications of mathematics to problems from the field of the natural sciences, geography, and the relations of human society are to be constantly made, though without endangering the independent importance of mathematics."

To sum up, I plead for the study of mathematics for the training and culture it contains because:

1. While recognizing its great practical value to the race it is only here and there we find an individual who will ever apply it (outside of the fundamentals of arithmetic) in a practical way. But, on the other hand, it furnishes a type of thought and training which is universal and used every day by every person.

2. Even in the matter of making a living the training and culture derived from mathematics furnishes an equipment which in value far outweighs, for most people, the direct practical value.

3. The spirit of the age is commercial, and to emphasize the utilitarian view in the schools is uncalled for, misleading, and undermining the morals in that its tendency and spirit is selfish and not altruistic; while the study of mathematics for training and culture—the study of mathematics for its own sake—leads to unselfishness and makes better citizens, and it is better citizens the schools must produce.

If properly studied and taught mathematics will develop all the powers of mind and heart, and one of the great needs of the world today is well trained minds and larger hearts—men and women who have high ideals and who are willing to live and work for humanity's sake.

There are hermit souls that live withdrawn
  In the place of their self-content;
There are souls like stars that dwell apart
  In a fellowless firmament;
There are pioneer souls that blaze a path
  Where highways never ran.
Let me live in a house by the side of the road
  And be a friend to man.

Let me live in a house by the side of the road,
  Where the race of men go by,—
The men that are good, the men that are bad,
  As good and as bad as I.
Then why should I sit in the scorner's seat,
  Or hurl the cynic's ban?
Let me live in a house by the side of the road
  And be a friend to man.

I see from my house by the side of the road,
  By the side of the highway of life,
The men that press on with the ardor of hope,
  And the men that are faint with the strife.

And I turn not away from their smiles and their tears,—
Both parts of an infinite plan.
Let me live in a house by the side of the road
And be a friend to man.

I know there are brook-gladdened meadows ahead,
And mountains of wearisome height,
That the road stretches on through the long afternoon
And passes away to the night.
Yet still I rejoice when the travelers rejoice,
And weep with the strangers that moan;
Nor live in my house by the side of the road
Like a man that lives alone.

Let me live in a house by the side of the road
Where the race of men go by.
They are good, they are bad, they are weak, they are strong,
Wise, foolish; so am I.
Then why should I sit in the scorner's seat
Or hurl the cynic's ban?
Let me live in a house by the side of the road
And be a friend to man.*

I plead for more rather than less of this great subject. I also plead for proper time to gain its results. We hear a whisper now and then that too much time is given to it for the results produced. This may in some instances be true, but that is not the fault of the subject. With good teaching and proper study and time devoted to it the results are assured.

Finally I plead for more teachers capable and willing to enter into this spirit of mathematics, and who are willing to work hard, for work hard they must to attain its results, and who will find pleasure in doing and inspire their students with the same spirit. When we have these and when we as teachers give our best efforts the general public will appreciate more than they do now the real value of mathematics.

There are loyal hearts, there are spirits brave,
There are souls that are pure and true;
Then give to the world the best you have,
And the best will come back to you.

* "The House by the Side of the Road," by Sam Walter Foss. Printed here by permission.

Give love, and love to your heart will flow,
A strength in your utmost need;
Have faith, and a score of hearts will show
Their faith in your word and deed.

For life is the mirror of king and slave,
'Tis just what you are and do;
Then give to the world the best you have,
And the best will come back to you.*

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THE AMERICAN WORK OF THE INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS.

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The Fourth International Congress of Mathematicians, held at Rome in 1908, adopted a resolution empowering Professors Klein of Göttingen, Sir George Greenhill of London, and Fehr of Geneva to form an international commission for the investigation of the teaching of mathematics in the secondary schools of the different nations, and to report to the next congress, which is to be held in England in 1912. This committee on organization met and took counsel as to the method of selecting the members of the commission, and finally decided that each of the countries represented by at least two delegates in at least two international congresses should have two or three delegates. They also decided that countries participating in the congresses but not having the required number of representatives should be entitled to one delegate, and that other countries likely to contribute valuable information should be invited to name a delegate who should act without vote. The committee on organization further decided that inasmuch as the term "secondary schools" had various meanings in different countries, the investigation should consider the teaching of mathematics in a broader sense, covering the entire field from the first steps through the courses required for any line of advanced

* Anonymous.